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On the response of a poro-elastic bed to water waves

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The problem of the response of a porous elastic bed to water waves is treated analytically on the basis of the three-dimensional consolidation theory of Biot (1941). Exact solutions for the pore-water pressure and the displacements of the porous medium are obtained in closed form for the case of waves propagating over the poro-elastic bed. The theoretical results indicate that the bed response to waves is strongly dependent on the permeability k and the stiffness ratio G/K', where G is the shear modulus of the porous medium and K' is the apparent bulk modulus of elasticity of the pore fluid. The earlier solutions for pore-water pressure by various authors are given as the limiting cases of the present solution. For the limits $G/K' \rightarrow 0$ or $k \rightarrow \infty$, the present solution for pressure approaches the solution of the Laplace equation by Putnam (1949). For the limit $G/K' \rightarrow \infty$, the present solution approaches the solution of the heat conduction equation by Nakamura *et al.* (1973) and Moshagen & Tørum (1975).

The theoretical results are compared with wave tank experimental data on pore-water pressure in coarse and fine sand beds which contain small amounts of air. Good agreement between theory and experiment is obtained.

1. Introduction

When water waves propagate over a porous bed such as a sand bed, fluid flow is induced in the porous medium and the porous medium itself is forced to deform. Thus the bed response to water waves is actually a combination of fluid and solid mechanical effects.

There have been numerous investigations of the problem of the flow induced in a porous bed by water waves, including Liu (1973), Massel (1976), Moshagen & Tørum (1975), Nakamura *et al.* (1973), Putnam (1949), Reid & Kajiura (1957), and Sleath (1970). However, they all assumed that the porous beds were rigid and non-deformable. In addition, all except Moshagen & Tørum (1975) and Nakamura *et al.* (1973) assumed that the pore fluid was incompressible. The fluid motion in the porous bed is usually expressed by Darcy's law which, with the assumption of a rigid bed with isotropic permeability and incompressible water, leads to the Laplace equation for the pore-water pressure. The consequence of this theory is that the pore-water pressure response

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is independent of the permeability of the bed material. Recently, a theory has been advanced by Massel (1976) to take into account nonlinear damping and the inertia term in the momentum equation in place of Darcy's law for rigid, porous beds. However, the conclusion from the theory was that the influence of the permeability on the pressure distribution in both sea and seabed is negligibly small and that the result is essentially the same as that from the Laplace equation.

The second approach taken by Nakamura *et al.* (1973) and Moshagen & Tørum (1975) is based on the assumption that the water is compressible while the porous bed is non-deformable, which leads to the heat conduction equation for the pore-water pressure. The conclusion from this assumption was that the pore-water pressure response is strongly dependent on the permeability of the bed material. The pressure attenuated rapidly and had a phase lag in fine soils. Nakamura *et al.* (1973) compared the theoretical results with the laboratory experiments on the pore pressure response in a fine sand bed and in a coarse sand bed. The experimental data showed unexplainable pressure discontinuities near the bed surface. Since the waves generated in their experiments were steep, the stress state in the sand beds under the wave crest and trough might have reached the limit equilibrium or the state of liquefaction, causing the large pressure drops (see Yamamoto 1977). The experimental results for the coarse sand showed no phase lag and agreed reasonably well with the solution from the Laplace equation. The data for the fine sand showed a large pressure attenuation and a large phase delay. Their calculations showed fairly good agreement with their experiments in both magnitude and phase lag. However, a critical error was found in their calculations. The compressibility of water used in the calculation was 980 times that of real water. It will be shown from the present theory that the false agreement reported may be explained by a small amount of air probably existing in the sand used.

On the other hand, the assumption common to the investigations concerned with the bed deformation from water waves such as Prevost *et al.* (1975) and Mallaid & Dalrymple (1977) is that the bed is an elastic continuum and no fluid flow takes place in the bed. This is a classical solid mechanical problem and the solution can be found in the text books on elasticity. Assuming that the pore-water pressure is equal to the change in the octahedral normal stresses in the elastic continuum, Prevost *et al.* (1975) concluded that the pore-water pressure is the same as the one obtained from the Laplace equation and, therefore, is independent of the permeability of the soil. This approach is, however, not physically consistent.

Pore-water flow, volume change, and deformation occur simultaneously in real soil beds. In order to take into account all of the effects, the analysis must be based on a more sophisticated mathematical model for the behaviour of the fluid/porous-medium complex. Biot (1941) presented a theory which takes into account the elastic deformation of the porous medium, the compressibility of pore fluid, and the Darcian flow of pore fluid. The purpose of this paper is to examine the bed response to water waves as a combination of the fluid and solid mechanical effects based on the Biot theory. The formulation of the equations presented in this paper is based on the original work by Koning (1968). It will be shown that all the earlier solutions for the pore-water pressure response obtained by Putnam (1949), Nakamura *et al.* (1973), and Prevost *et al.* (1975) are indeed the three extreme cases of the more general solution presented in this paper. The theoretical results will be compared with the pore-water pressure response in sand beds measured in laboratory experiments.

2. Governing equations

The proposed model is based on the physically consistent, three-dimensional consolidation theory developed by Biot (1941). Since a complete and clear-cut derivation of the theory is given in the original paper, as well as in many text books (e.g. Verruijt 1969), it will not be repeated here. The basic assumption is that the soil skeleton obeys Hooke's law, i.e. that the soil has linear, reversible, isotropic, non-retarded mechanical properties. Since we are interested in the relatively small oscillatory deformation relative to the hydrostatic equilibrium state, such an idealized assumption may be reasonable. The movement of the pore fluid is assumed to obey Darcy's law.

The problem considered in this paper is two-dimensional. An infinitely deep, homogeneous isotropic sediment is considered. The x axis is taken on the bed surface and the positive direction of the z axis is taken vertically downward from the bed surface. The waves travel from right to left.

Since the hydrostatic components of the pore-water pressure, stresses, and strains in the soil are trivial to the following consideration of the problem, only the incremental components of such variables will be considered unless otherwise mentioned.

The continuity equation is given by

$$\frac{k}{\gamma}\nabla^2 p = \frac{n}{K'}\frac{\partial p}{\partial t}+\frac{\partial \epsilon}{\partial t}, \tag{2.1}$$

in which p is the excess pore-water pressure, ϵ is the volume strain of the porous medium, t is the time, k is the coefficient of permeability of the soil, γ is the unit weight of the pore-water, n is the porosity, and K' is the apparent bulk modulus of pore-water.

If the pore-water is absolutely air-free, K' is equal to the true bulk modulus of elasticity of water K. However, if the pore-water contains even a very small amount of air, the apparent bulk modulus of elasticity of the water decreases drastically and K' is related to K by (Verruijt 1969)

$$\frac{1}{K'} = \frac{1}{K}+\frac{1-S_r}{P_0}, \tag{2.2}$$

where S_r is the degree of saturation and P_0 is the absolute pore-water pressure. The volume strain for the two-dimensional problem is

$$\epsilon = \frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}, \tag{2.3}$$

where u is the x component of soil displacement and w is the z component of soil displacement.

From the effective stress concept and Hooke's law, the equations of equilibrium are

$$G\nabla^2 u+\frac{G}{1-2\nu}\frac{\partial \epsilon}{\partial x} = \frac{\partial p}{\partial x}, \tag{2.4}$$

$$G\nabla^2 w+\frac{G}{1-2\nu}\frac{\partial \epsilon}{\partial z} = \frac{\partial p}{\partial z}, \tag{2.5}$$

where ν is Poisson's ratio for the soil, G is the shear modulus of the soil, and G is related to Young's modulus E and ν by

$$G = \frac{E}{2(1+\nu)}. \tag{2.6}$$

The effective stresses are related to the strains by Hooke's law as

$$\sigma'_x = 2G\left[\frac{\partial u}{\partial x} + \frac{\nu}{1-2\nu}\epsilon\right], \tag{2.7}$$

$$\sigma'_z = 2G\left[\frac{\partial w}{\partial z} + \frac{\nu}{1-2\nu}\epsilon\right], \tag{2.8}$$

$$\tau_{xz} = G\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right], \tag{2.9}$$

where σ'_x is the effective normal stress in the x direction, σ'_z is the effective normal stress in the z direction, and τ_{xz} is the shear stress in the z direction on the plane perpendicular to the x axis.

Equations (2.1), (2.4) and (2.5) form a system of three partial differential equations in terms of the three unknown variables, p, u and w, to be solved for particular boundary conditions.

3. Boundary-value problem

In this section, the three simultaneous partial differential equations will be solved for the case of waves propagating over a porous bed. At the top of the bed ($z = 0$) the pressure fluctuates owing to surface waves. The pressure fluctuation attenuates as the waves travel over the bed owing to the energy loss in the bed. However, the attenuation rate is usually small and may be neglected when only the region around a structure is considered. Thus the pressure at the bed surface is assumed to be periodic in this development. Its value may be determined by experiment or, ignoring the damping, by higher-order wave theories. In any event, the periodic signal can be expanded in a Fourier series and it is, therefore, sufficient to study a sinusoidal fluctuation.

3.1. *Boundary conditions*

In order to solve the three simultaneous partial differential equations, one needs three independent conditions per boundary. At the bed surface, the boundary conditions are that the vertical effective stress is zero, that the shear stress is negligibly small, and that the sinusoidal pressure fluctuation exists, or at $z = 0$:

$$\sigma'_z = 2G\left[\frac{\partial w}{\partial z} + \frac{\nu}{1-2\nu}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right] = 0, \tag{3.1a}$$

$$\tau_{xz} = G\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0, \tag{3.1b}$$

$$p = p_0 \exp\left[i(\lambda x + \omega t)\right], \tag{3.1c}$$

where p_0 is the amplitude of pressure fluctuation at the bed surface, λ is the wave-number, and ω is the angular wave frequency, and only the real part is considered in the last equation.

The boundary conditions for a semi-infinite half-plane may be given as

$$u, w, p \to 0 \quad \text{as} \quad z \to \infty. \tag{3.2}$$

3.2. *Harmonic solutions*

Since the boundary condition (3.1) is periodic in both time and space, it is reasonable to assume that u, w and p are also periodic in time and space, or

$$u = U(z) \exp[i(\lambda x + \omega t)], \tag{3.3a}$$

$$w = W(z) \exp[i(\lambda x + \omega t)], \tag{3.3b}$$

$$p = P(z) \exp[i(\lambda x + \omega t)], \tag{3.3c}$$

in which only the real parts are considered, as before, and U, W and P are functions of z only.

Substitution of (3.3a, b, c) into the three governing partial differential equations (2.1), (2.4) and (2.5) leads to three simultaneous ordinary differential equations of second order. The differential equations are linear and homogeneous and the solutions can be found by forming the characteristic equation of the operator, $D = d/dz$. One will find the characteristic equation as

$$(D^2 - \lambda^2)^2 (D^2 - \lambda'^2) = 0, \tag{3.4}$$

where

$$\lambda'^2 = \lambda^2 + i \frac{\gamma}{k} \omega \left[\frac{n}{K'} + \frac{(1-2\nu)}{2(1-\nu) G} \right]. \tag{3.5}$$

The characteristic equation has two equal roots $+\lambda$, two equal roots $-\lambda$, and two simple roots $\pm \lambda'$. Hence the general solutions are

$$U = a_1 \exp(\lambda z) + a_2 \exp(-\lambda z) + a_3 z \exp(\lambda z) + a_4 z \exp(-\lambda z) + a_5 \exp(\lambda' z) + a_6 \exp(-\lambda' z), \tag{3.6a}$$

$$W = b_1 \exp(\lambda z) + b_2 \exp(-\lambda z) + b_3 z \exp(\lambda z) + b_4 z \exp(-\lambda z) + b_5 \exp(\lambda' z) + b_6 \exp(-\lambda' z), \tag{3.6b}$$

$$P = c_1 \exp(\lambda z) + c_2 \exp(-\lambda z) + c_3 z \exp(\lambda z) + c_4 z \exp(-\lambda z) + c_5 \exp(\lambda' z) + c_6 \exp(-\lambda' z), \tag{3.6c}$$

where a_n, b_n and c_n ($n = 1, \ldots, 6$) are constants which have to be determined from the boundary conditions and the governing equations (2.1), (2.4) and (2.5).

From the semi-infinite half-plane boundary conditions (3.2), the constants a_n, b_n and c_n ($n = 1, 3, 5$) vanish and (3.6a, b, c) become

$$U = a_2 \exp(-\lambda z) + a_4 z \exp(-\lambda z) + a_6 \exp(-\lambda' z), \tag{3.7a}$$

$$W = b_2 \exp(-\lambda z) + b_4 z \exp(-\lambda z) + b_6 \exp(-\lambda' z), \tag{3.7b}$$

$$P = c_2 \exp(-\lambda z) + c_4 z \exp(-\lambda z) + c_6 \exp(-\lambda' z). \tag{3.7c}$$

The coefficients a_n, b_n and c_n are not independent. The dependence can be determined by substitution of (3.7a, b, c) into (2.1), (2.4) and (2.5), i.e. b_n and c_n can be given in terms of a_n. By substitution of the constants b_n and c_n so determined into (3.7a, b, c) one obtains

$$U = a_2 \exp(-\lambda z) + a_4 z \exp(-\lambda z) + a_6 \exp(-\lambda' z), \tag{3.8a}$$

$$W = i\left[a_2 + \frac{a_4}{\lambda}\frac{1+(3-4\nu)m}{1+m}\right]\exp(-\lambda z) + i a_4 z \exp(-\lambda z) + i\frac{\lambda}{\lambda'} a_6 \exp(-\lambda' z), \tag{3.8b}$$

$$P = i\frac{2G}{1+m} a_4 \exp(-\lambda z) + \frac{2a_6}{\lambda}\omega' \beta G \exp(-\lambda' z), \tag{3.8c}$$

in which

$$\beta = (1-\nu)/(1-2\nu), \tag{3.9}$$

$$m = \frac{n}{K'}\frac{G}{1-2\nu}, \tag{3.10}$$

$$\omega' = \omega/c, \tag{3.11}$$

$$c = \frac{k}{\gamma}\bigg/\left[\frac{n}{K'} + \frac{1-2\nu}{2(1-\nu)G}\right]. \tag{3.12}$$

The three constants a_2, a_4 and a_6 can be determined from the boundary conditions (3.1a, b, c) at the bed surface. Finally, we obtain the exact solutions for u, w and p as

$$u = i\left\{m\frac{-[1+2(1-\nu)\lambda''] + i(1-2\nu)\omega''}{-\lambda'' + i(1+m)\omega''}\exp(-\lambda z) - \left[1 - \frac{m\lambda''}{-\lambda'' + i(1+m)\omega''}\right]\right.$$
$$\left.\times \lambda z \exp(-\lambda z) + \frac{m}{-\lambda'' + i(1+m)\omega''}\exp(-\lambda' z)\right\}\frac{p_0}{2\lambda G}\exp[i(\lambda x + \omega t)], \tag{3.13a}$$

$$w = \left\{\left[1 + m\frac{1+(1-2\nu)(-\lambda'' + i\omega'')}{-\lambda'' + i(1+m)\omega''}\right]\exp(-\lambda z) - \left[1 - \frac{m\lambda''}{-\lambda'' + i(1+m)\omega''}\right]\right.$$
$$\left.\times \lambda z \exp(-\lambda z) - \frac{m(1+\lambda'')}{-\lambda'' + i(1+m)\omega''}\exp(-\lambda' z)\right\}\frac{p_0}{2\lambda G}\exp[i(\lambda x + \omega t)], \tag{3.13b}$$

$$p = \left\{\left[1 - \frac{im\omega''}{-\lambda'' + i(1+m)\omega''}\right]\exp(-\lambda z) + \frac{im\omega''}{-\lambda'' + i(1+m)\omega''}\exp(-\lambda' z)\right\}$$
$$\times p_0 \exp[i(\lambda x + \omega t)], \tag{3.13c}$$

where

$$\omega'' = \beta(\omega'/\lambda^2), \tag{3.14}$$

$$\lambda'' = (\lambda' - \lambda)/\lambda. \tag{3.15}$$

In order to appreciate the physical significance of the exact solutions (3.13a, b, c) two special cases are considered in the following.

3.3. *Completely saturated soils* ($G/K' \to 0$)

If the soil is completely saturated with water and if the pore-water does not contain gases during the entire cycle, then the apparent modulus of elasticity, K', is equal to the true modulus of elasticity of water, $K = 4 \times 10^7$ p.s.f. ($1{\cdot}9 \times 10^9\,\mathrm{N/m^2}$). Since the value of G for soils varies from about 10^7 p.s.f. ($4{\cdot}8 \times 10^8\,\mathrm{N/m^2}$) for very dense sand to 10^4 p.s.f. ($4{\cdot}8 \times 10^5\,\mathrm{N/m^2}$) for silt and clay, the stiffness ratio, G/K', becomes

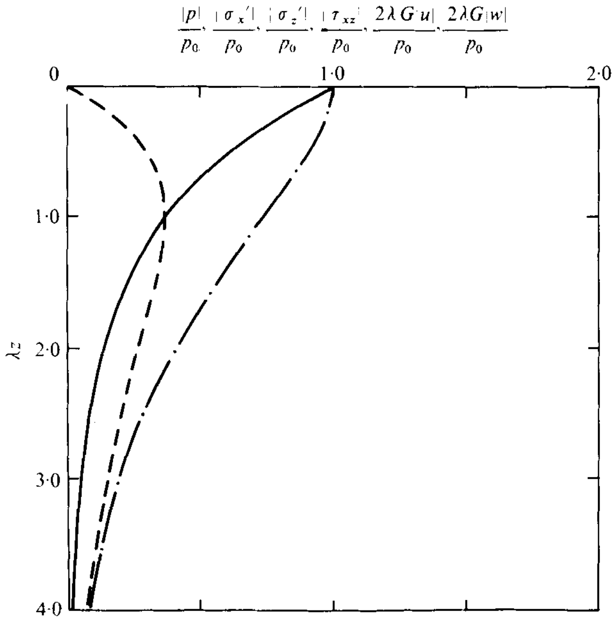

FIGURE 1. Vertical distribution of the amplitudes the pore-water pressure, the effective stresses, and the displacements for the limiting case $G/K' \to 0$. ——, $|p|/p_0$; —·—, $2\lambda G|w|/p_0$; – – –, $2\lambda G|u|/p_0$, $|\sigma_x'|/p_0$, $|\sigma_z'|/p_0$, $|\tau_{xz}|/p_0$.

practically zero for most soils except for dense sand. As $G/K' \to 0$, $m \to 0$, for the limit we obtain from (3.13)

$$u = -i\lambda z \exp(-\lambda z)(p_0/2\lambda G)\exp[i(\lambda x+\omega t)], \tag{3.16a}$$

$$w = [\exp(-\lambda z)+\lambda z\exp(-\lambda z)](p_0/2\lambda G)\exp[i(\lambda x+\omega t)], \tag{3.16b}$$

$$p = p_0\exp(-\lambda z)\exp[i(\lambda x+\omega t)]. \tag{3.16c}$$

It is interesting to note that the pore-water pressure response for this case is the same as that obtained by Putnam (1949), who assumed that the soil is rigid and water is incompressible, and that obtained by Prevost *et al.* (1975), who assumed that the soil is an elastic continuum and no fluid flow takes place in the soil. The pressure attenuation for this case is small and independent of the permeability of soil. As can be seen from (3.16), however, such good transmission of the pressure has to be associated with the deformation of the soil. The amplitudes of the displacements and the pore-water pressure after non-dimensionalization are plotted in figure 1. A given soil particle moves on an elliptical orbit in general. Near the bed surface the motion is only vertical. For $\lambda z > 4{\cdot}0$, the orbit becomes essentially a circle.

The effective stresses for this case can be obtained by substituting (3.16) into (2.7), (2.8) and (2.9), and they are given as

$$\sigma_x' = -\sigma_z' = -i\tau_{xz} = p_0\lambda z\exp(-\lambda z)\exp[i(\lambda x+\omega t)]. \tag{3.17}$$

Substituting (3.16) into (2.3), one finds that the volume strain, ϵ, is always zero for this case – no volume change. The maximum absolute values of the stresses given by (3.17) are also plotted in figure 1. All three effective stresses increase from zero at the bed surface ($z = 0$) to the maximum value, $0{\cdot}36p_0$ at $\lambda z = 1$, and then gradually decrease as z is increased.

If the permeability, k, of the soil is large, ω'' becomes small and (3.13*c*) tends to (3.16*c*) even for a slightly unsaturated case. This will be further discussed in the experiments section.

3.4. *Partially saturated dense sand and sandstone* ($G/K' \to \infty$)

Another interesting case is when the stiffness of the soil becomes much larger than that of pore-fluid, or $G/K' \to \infty$. Physical examples will be sandstone and very dense sand [$G = 10^7$ p.s.f. ($4{\cdot}8 \times 10^8\,\mathrm{N/m^2}$)] saturated with a mixture of liquid and gas. If the sand is 95 % saturated with water at atmospheric pressure, the apparent modulus of elasticity is, from (2.2), $K' = 2 \times 10^4$ p.s.f. ($9{\cdot}5 \times 10^5\,\mathrm{N/m^2}$) or $G/K' = 1250$.

For the limit $G/K' \to \infty$, (3.13) becomes

$$u = i\left\{\left[(1-2\nu)+\frac{i}{\omega''}(1+2(1-\nu)\lambda'')\right]\exp(-\lambda z)-\left(1+i\frac{\lambda''}{\omega''}\right)\lambda z\exp(-\lambda z) - \frac{i}{\omega''}\exp(-\lambda' z)\right\}\frac{p_0}{2\lambda G}\exp[i(\lambda x+\omega t)], \quad (3.18a)$$

$$w = \left\{\frac{1+(1-2\nu)(-\lambda''+i\omega'')}{i\omega''}\exp(-\lambda z)+\left(1+\frac{i\lambda''}{\omega''}\right)\lambda z\exp(-\lambda z) + \frac{i}{\omega''}(1+\lambda'')\exp(-\lambda' z)\right\}\frac{p_0}{2\lambda G}\exp[i(\lambda x+\omega t)], \quad (3.18b)$$

$$p = \exp(-\lambda' z)\,p_0\exp[i(\lambda z+\omega t)]. \quad (3.18c)$$

The pressure equation (3.18*c*) is essentially the same as that obtained by Nakamura *et al.* (1973) and Moshagen & Tørum (1975), assuming a rigid soil and compressible fluid. Since $|\lambda'| > \lambda$ for this case, the pressure attenuates rapidly compared with (3.13*c*) and there is a phase delay in the pore pressure response which increases linearly as z is increased. However, the attenuation of displacements and, thus of the stresses, is slow as can be seen from (3.18*a, b*).

Depending on the stiffness and permeability of the bed material and the gas content in the pore-water, the transmission of pressure, stresses and deformation in the sediment falls somewhere in between the two extreme cases just considered and thus must be determined by the exact solution (3.13*a, b, c*).

The theoretical pressure equation (3.13*c*) will be compared with the experimental data obtained from laboratory tests in the next section.

4. Experiments

The pore pressure measurements were made for both coarse and fine sands at the Delft Hydraulics Laboratory. The experimental set-up is shown in figure 2. The thickness of the sand bed, d, was 50 cm and the water depth 90 cm. The wave period, T, was varied from 1 to 2·6 seconds. The coarse and fine sands were both fairly uniform and the average grain sizes were 1·2 mm and 0·2 mm, respectively. The pore pressures at various vertical distances from the bed surface were measured by five pore-water pressure transducers. In order to avoid the sand drifting at the surface and the

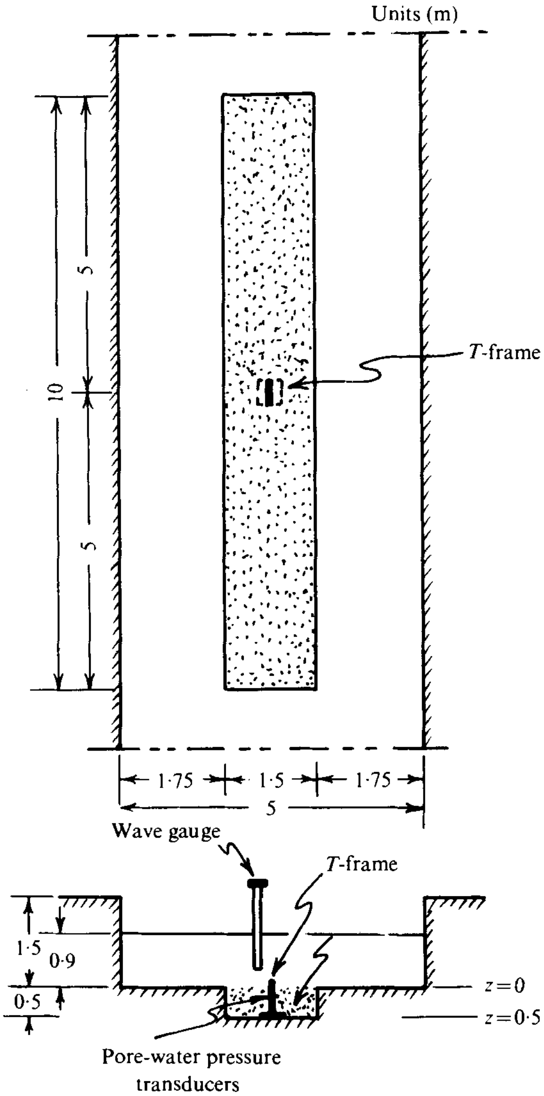

FIGURE 2. Experimental set-up for the pore-water pressure measurements in the sand bed in the wave tank. The units are in metres.

liquefaction of the sand bed by waves, the wave height and the wave steepness were kept small.

5. Comparisons of theory and experiment

The data for the coarse sand are plotted in figure 3. There was practically no phase shift in these data. The data agree very well with the theoretical values determined by the solution of the Laplace equation for finite depth given by Putnam (1949); this is because the permeability, k, of the soil is so large that ω'' becomes small, rather than because the sand is completely saturated as discussed in the previous section.

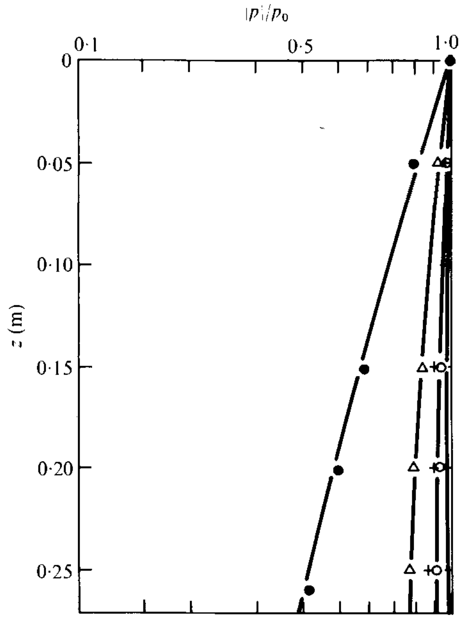

FIGURE 3. Vertical distribution of the amplitude of the pore-water pressure in the coarse sand. ——, present theory and the theory by Putnam (1949). Experimental data: ●, $T = 1{\cdot}0$ s; △, 1·5; +, 2·0; ○, 2·6.

For this case, the pressure equation (3.13*c*) approaches to the solution by Putnam (1949).

Typical simultaneous recordings of wave and pore pressures at various depths for the fine sand are given in figure 4. As z increases, the pore pressure decreases and a phase delay appears.

Since a direct determination of k, G, ν and K' was not made, direct comparisons between the theory and the experiments cannot be done. However, some indirect comparisons are made in the following.

From the three series of data carried out for different wavelengths, the best fit, in the least-squares sense, to (3.13*c*) yielded the following values: $\nu = 1/3$, $c = 0{\cdot}02\ \mathrm{m^2/s}$ and $m = 1$.

The air content estimated from (2.2) was about 2 % by volume. The direct measurements of air contents for a similar sand at similar experimental conditions revealed a value of 2 % (Nath *et al.* 1977). This supports the credibility of the estimated values of ν, c and m in some degree.

The measured amplitudes of the excess pore-water pressure and the results of the calculations with (3.13*c*) are shown in figure 5. Generally, good agreement between the present theory and experimental data is shown.

In figure 6, the experimental data for $T = 2$ s are compared with various theories. In these calculations the determined value of $c = 0{\cdot}02\ \mathrm{m^2/s}$ is used. The value of d has hardly any influence on the results obtained from the heat conduction equation by Moshagen & Tørum (1975) and Nakamura *et al.* (1973), whereas the results obtained from the Laplace equation by Putnam (1949) for $d = 0{\cdot}5$ m differ from those for $d \to \infty$.

The theoretical values calculated by (3.13*c*) of the present theory lie between the

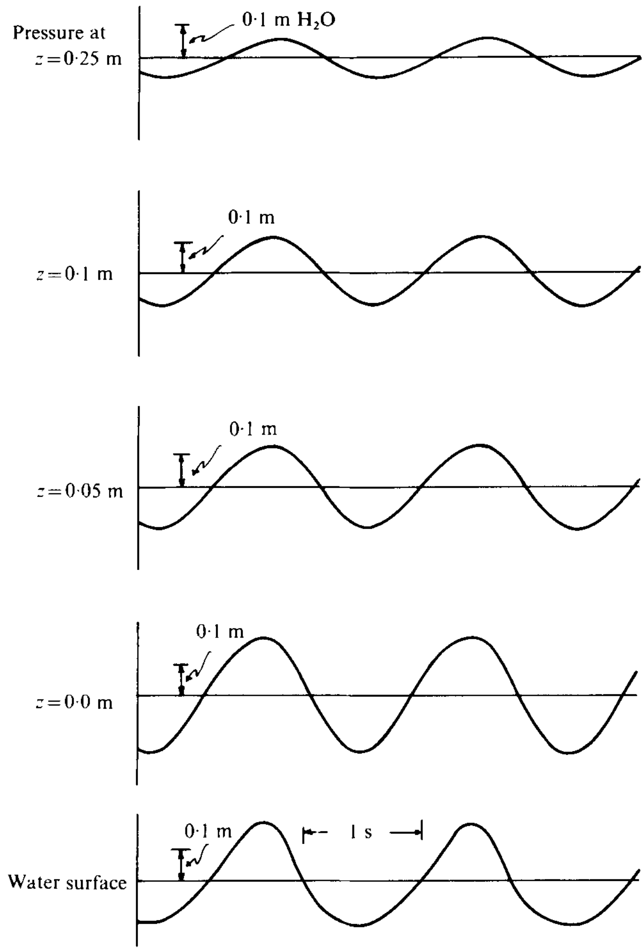

FIGURE 4. Example of recordings of the pore-water pressure and the waves for the fine sand; $T = 2{\cdot}0$ s.

solutions of the Laplace equation and the heat conduction equation, but closer to those of the heat conduction equation for this condition. Thus, the effect of the bottom boundary is probably small. The comparisons of theories and experimental data for the phase delay are shown in figure 7. Generally, good agreement is obtained between the data and present theory.

As demonstrated here, none of the previously proposed theories are adequate for predicting the pore-water pressure generation in the fine sand. The Laplacian solution by Putnam (1949) predicts too small a pressure attenuation in soils, and the solution of the heat conduction equation by Moshagen & Tørum (1975) and Nakamura *et al.* (1973) predicts too large a pressure attenuation. Furthermore, neither of these provides information on the stress and strain in soils. The present theory seems to agree very well with the laboratory experiments. The theory provides the information on the stress–strain state in soils.

Further stress analysis based on the present theory indicates that the soils can be liquefied by passage of a steep wave (Yamamoto 1977).

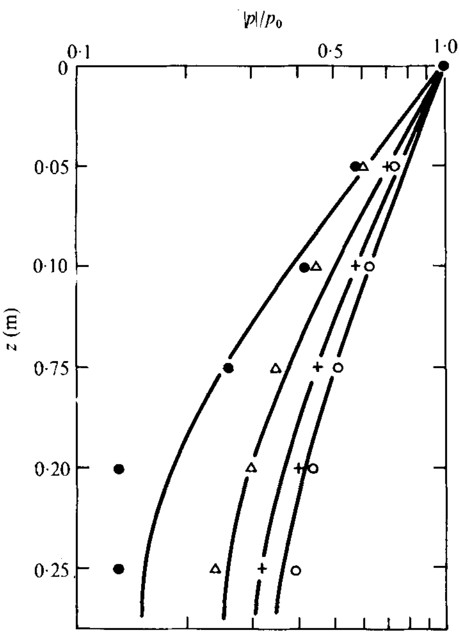

FIGURE 5

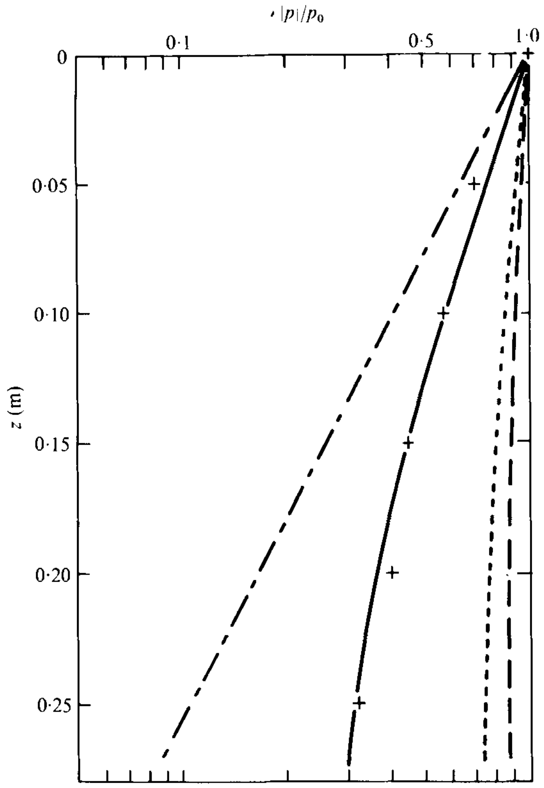

FIGURE 6

FIGURE 5. Vertical distribution of the amplitude of the pore-water pressure in the fine sand. ——, present theory. Experimental data: ●, $T = 1{\cdot}0$ s; △, 1·5; +, 2·0; ○, 2·6.

FIGURE 6. Comparison between various theories and experiments for the amplitude of the pore-water pressure in the fine sand; $T = 2{\cdot}0$ s. ——, present theory ($d = \infty$); —·—, the theory by Moshagen & Tørum (1975) ($d = 0{\cdot}5$ m, ∞); ---, the theory by Putnam (1949) ($d = 0{\cdot}5$ m). Experimental data are shown by +.

6. Summary and conclusions

We have obtained the exact closed-form solutions for the pore-water pressure, the displacements, and the effective stresses in the elastic porous bed induced by water waves. The bed response is influenced by the permeability, the stiffness of the porous medium, and the compressibility of the pore fluid. The earlier solutions for the pore-water pressure response by Putnam (1949), Nakamura *et al.* (1973), Moshagen & Tørum (1975) and Prevost *et al.* (1975) are all given as extreme cases of the present solution.

When the stiffness of the porous medium is much smaller than that of the pore fluid, such as for saturated soft soils, the bed response becomes independent of the permeability and has no phase lag. The pressure response approaches the solution by Putnam (1949) for a rigid bed and incompressible pore fluid and the solution by Prevost *et al.* (1975) for an elastic continuum without pore fluid flow. However,

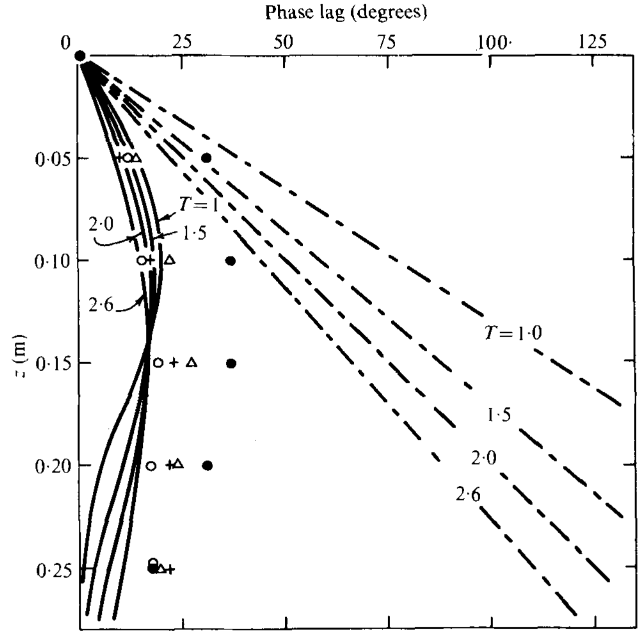

FIGURE 7. Vertical distribution of the phase lag in the pore-water pressure response in the fine sand. ——, present theory ($d = \infty$); —·—, the theory by Moshagen & Tørum (1975) ($d = 0{\cdot}5$ m, ∞). Experimental data: ●, $T = 1{\cdot}0$ s; △, 1·5; +, 2·0; ○, 2·6.

the good pressure transmission is associated with a good deformation of the porous medium.

On the other hand, when the stiffness of the porous medium is much larger than that of the pore fluid, such as for partially saturated dense sands, the pressure response approaches the solutions by Nakamura *et al.* (1973) and Moshagen & Tørum (1975) for a rigid porous bed and compressible pore fluid as expected. The pressure attenuates rapidly and the phase lag increases linearly as the distance from the bed surface is increased. However, the stresses in the porous medium attenuate slowly for this case.

Depending on the permeability, the stiffness of the porous medium, and the compressibility of the pore fluid, the bed response to water waves falls somewhere between the two extremes and, thus, the present solution should be used.

The theoretical results have been compared with the results from laboratory experiments on the pore-water pressures in the coarse and fine sands. Good agreement between the present theory and the experiment has been obtained.

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